XI Congress GAFEVOL

October 23-26, 2017

Instituto de Exatas
Departamento de Matemática
Universidade de Brasília

Brasília, BRAZIL

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Welcome

It is with great honor that we welcome you to the XI Congress GAFEVOL with will take place at the University of Braslia, Brazil, on October 23-26,2017. We wish you a pleasent stay and that you like the congress.

Organizing Committee

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Address

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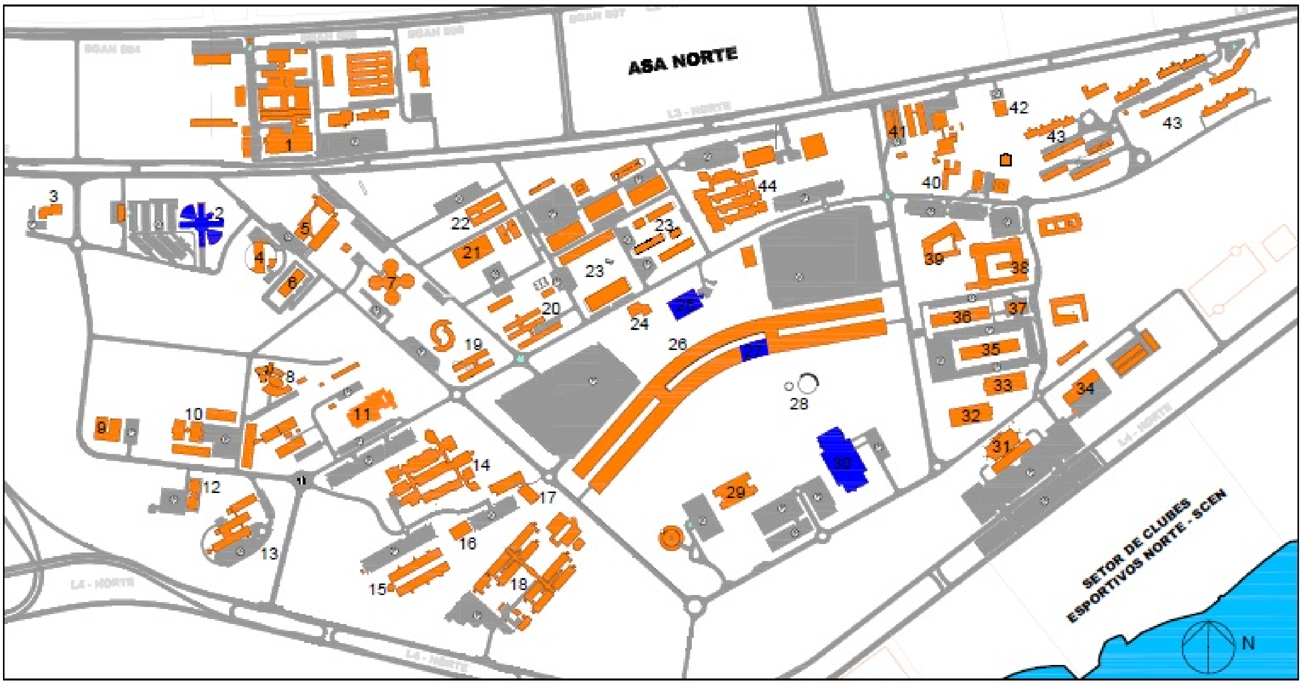
Map of the University of Brasília



1- HUB (Hospital Universitário) - University hospital	16- MASC	31- Centro Comunitário Athos Bulcão - Community Center
2- Finatec	17- BSA	32- CIC
3- FUBRA	18- Instituto de Biologia - Biology Institute	33- UED
4- AUTOTRAC	19- PMU II	34- Almoxarifado - Warehouse
5- FIO CRUZ	20- PMU II	35- PJC
6- CAEP	21- FE	36- PAT
7- CET	22- Casa do Professor	37- MASC
8- CRAD	23- SG 1 ao SG 12	38- FACE
9- CDT	24- Banco do Brasil - Bank	39- FA
10- CME	25- Restaurante Universitário - University Restaurant	40- ASFUB
11- NMT	26- ICC	41- Posto de Gasolina/Subway/Spoletto - Gas Station/Restaurant
12- CPD	27- Departamento de Matemática - Math departament	42- PMDF - Police
13- CESPE	28- Teatro de Arena Honestino Guimarães - Arena Theater	43- Colina
14- Faculdade de Saúde - Health College	29- Reitoria - Rectory	44- FT
15- Instituto de Qimica - Chemistry Institute	30- Biblioteca - Library	P- Estacionamento - Parking

Contents

General Information

Location

The congress will take place in FINATEC at the University of Brasília. Also, the participant may want to know the departament of mathematics of the University of Brasília. Both places are indicated in blue in the map on page 4.

Useful Phone Numbers

In case of any health emergencies call 192 (SAMU).
Police number: 190.
Math department of the University of Brasília: (61) 3107 7236
Security of the University of Brasília: (61) 3107 6222

Meals and refreshments

There is a university restaurant indicated in the map of page 4 that serves breakfast (7:00hs - 9:00hs), lunch (11:00hs - 14:30hs) and dinner (17:00hs - 19:30hs). There are several restaurants nearby the in Asa Norte. We will present you a few options:
1. Feitio Mineiro located at 306 Norte, bloco B - lojas 45/51, Asa Norte (lunch and dinner).
2. Restaurante e Bar Xique Xique located at 708 Norte Bloco E Loja 45, Asa Norte (lunch and dinner).
3. Subway/Spoleto at the University of Brasília, indicated in the map of page 4, number 41 (lunch and dinner).
4. Domino's Pizza located at 109 Norte Bloco B, Loja 1, Asa Norte (Dinner).
5. Crepe au Chocolat located at 109 Norte, Bloco C, Loja 5, Asa Norte (Launch and Dinner).
6. Restaurante El Negro located at 413 Norte Bloco C, Loja 21, Asa Norte.
There is also a supermarket (Pão de Açucar) at 404/405 Norte, Bloco A, Asa Norte opened from 7:00hs to 22:00hs.

Schedule

Schedule	Monday October 23	Tuesday October 24	Wednesday October 25	Thursday October 26
8:40 – 9:10	Opening	Tomás Caraballo	Hans-Otto Walther	Martin Bohner
9:10 – 9:40		Alexandre Nolasco	Eduardo Hernández	Gabriela Planas
9:40 – 10:10	Carlos Lizama	Antonio Luiz Pereira	Rolando Rebolledo	María Pilar Velasco
10:10 - 10:40	Hernán Henríquez	Ma To Fu	Márcia Federson	Anne Bronzi
10:40 - 11:10	Coffee-break	Coffee-break	Coffee-break	Coffee-break
11:10 – 11:40	Aldo Pereira Solis	Marcone Pereira	Miguel Frasson	Igor Freire
11:40 – 12:10	Piotr Kalita	Everaldo Bonotto	Marta Gadotti	Andrea Arita
12:10 – 12:40	Phillipo Lappicy	Felipe Rivero	Marcos Napoleão Rabelo	Samuel Castillo
12:40 – 13:10	Alberto Mercado	Antonín Slavík	Vanessa Rolnik	Joelma Azevedo
13:10 – 14:30	Lunch	Photo & Lunch	Lunch	Lunch
14:30 – 15:00	Matheus Bortolan		Katia Azevedo	Filipe Andrade
15:00 – 15:30	Rodolfo Collegari	Tour	Fernando Gomes	Rodrigo Ponce
15:30 – 16:00	Paulo M. de Carvalho		Marielle Aparecida	Juliana Pimentel
16:00 – 16:30	Coffee-break & Poster Session		Coffee-break & Poster Session	Coffee-break
16:30 – 17:00				Salomón Alarcón
17:00 – 17:30	Ricardo Teles		Fernanda de Andrade	Philippe Caillol
17:30 – 18:00	Alexandre Nascimento		Rogélio Grau Acuña	Closing
18:00 – 18:30	Cocktail			
18:30 – 19:00				
19:00 – 19:30				
19:30 – 20:00				
20:00			Conference Dinner	

Abstracts of the Talks

Controllability of evolution PDEs by spectral methods

Alberto Mercado Saucedo (alberto.mercado@usm.cl)
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Abstract

In this talk we will introduce the moment method for controllability of PDEs, which is based on the properties of exponential functions related with the eigenvalues of the involved equations. We will present some recent results, obtained using this method, for Kuramoto Sivashinsky (KS) system, a parabolic fourth order partial differential equation, and other ongoing and related problems.

References

[1] Cerpa, E., Guzmn, P., Mercado, A. *On the control of the linear Kuramoto-Sivashinsky equation.* ESAIM Control Optim. Calc. Var. 23 (2017), no. 1, 165-194.

Approximate controllability via properties on resolvent operators

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Abstract

This talk treats the approximate controllability of fractional differential systems of Sobolev type in Banach spaces. We first characterize the properties on the norm continuity and compactness of some resolvent operators (also called solution operators). And then, via the obtained properties on resolvent operators and fixed point technique, we give some approximate controllability results for Sobolev type fractional differential systems in the Caputo and Riemann-Liouville fractional derivatives with order $1 < \alpha < 2$, respectively. This, in contrast to the development in [2] and [3], follows in part the idea of [1] to suppose certain properties on the resolvent operators to obtain approximate controllability, namely, the norm continuity and compactness. Particularly, the existence or compactness of an operator E^{-1} is not necessary in our results.

References

[1] Z. Fan. *Approximate controllability of fractional differential equations via resolvent operators.* Adv. Diff. Equat. **2014** (2014), no. 54, 11 pp.

[2] H. Qin, Z. Zuo, J. Liu, L. Liu. *Approximate controllability and optimal controls of fractional dynamical systems of order* $1 < q < 2$ *in Banach spaces.*. Adv. Difference Equ. **2015** (2015), no. 73, 17 pp.

[3] R. Sakthivel, R. Ganesh, Y. Ren, S. Anthoni. *Approximate controllability of nonlinear fractional dynamical systems.* Commun. Nonlinear Sci. Numer. Simul. **18** (2013), no. 12, 34983508.

Non-Autonomous Morse-Smale Dynamical Systems: Structural Stability under Non-Autonomous Perturbations

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Abstract

In this lecture we present our recent results on structural stability of gradient Morse-Smale Dynamical Systems under non-autonomous perturbations. To that end we introduce the notion of lifted invariant sets and give a characterization of the uniform attractor in terms of dynamical structures of a family of pullback attractors. This is a joint work with G. Raugel (Paris XI), J. Langa (U. Sevilla) and M. Bortolan (UFSC-Brazil).

Navier-Stokes equations: the one million dollar problem from the point of view of continuation of solutions

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Abstract

We consider the Navier-Stokes problem on $\mathbb{R}^N$

$$\begin{array}{l} u_t = \Delta u - \nabla \pi + f(t) - (u \cdot \nabla) u, \quad x \in \Omega \\ \operatorname{div}(u) = 0, \quad x \in \Omega \\ u = 0, \quad x \in \partial\Omega \\ u(0,x) = u_0(x), \end{array} \tag{1}$$

where $u_0 \in L^N(\Omega)^N$ and Ω is an open, bounded and smooth subset of $\mathbb{R}^N$. We prove that the above problem is locally well posed and give conditions to obtain that these solutions exist for all $t \geq 0$. We used techniques of semilinear parabolic equations considering nonlinearities with critical grouth developed in [1].

This work is based on my master's degree dissertation, and is not recent work, however we present a new interpretation for the problem above.

References

[1] Arrieta, J. and Carvalho, A. N., Abstract Parabolic Problems with Critical Nonlinearities and Applications to Navier-Stokes and Heat Equations. *Transactions of the American Mathematical Society*, **352** 285-310 (2000).

Existence of solutions for the aggregation equations with initial data in Morrey spaces

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Abstract

In this work we consider a class of nonlinear viscous transport equations describing aggregation phenomena in biology, which can be written in the form

$$u_t = \Delta u - \nabla \cdot (u(\nabla K * u)),\ x \in \mathbb{R}^n,\ t > 0, \tag{2}$$

$$u(x,0) = u_0(x),\ x \in \mathbb{R}^n, \tag{3}$$

where the unknown variable $u = u(x,t) \geq 0$ represents either the population density of a species or the density of particles in a granular media, $n \geq 2$, the Kernel $\nabla K \in L^1(\mathbb{R}^n)$ and the symbol "$*$" denotes the convolution with respect to the variable x.

When the initial data u_0 belongs to a Morrey space, for suitable indices, we show the existence of a global mild solution to (2)-(3). We also analyze the asymptotic stability of solutions persistence at large times.

References

[1] T. Kato. *Strong solutions of the Navier-Stokes equations in Morrey spaces* . Bol. Sol. Bras. Mat., **22**, 2: 127-155, 1992.

[2] H. Kozono and Y. Sugiyama. *Local existence and finite blow-up of solutions in the 2-D Keller-Segel system.* J. Evol. Equ., 8: 353-378, 2008.

[3] D. Li and X. Zhang. *On a nonlocal aggregation model with nonlinear diffusion.* Discrete Cont. Dyn. Syst., 27: 301-323, 2010.

On the Convergence of Statistical Solutions of Evolution Equations

Anne Bronzi (annebronzi@ime.unicamp.br)
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Abstract

In this talk we will present an abstract framework for the theory of statistical solutions for general evolution equations. This theory extends the notion of statistical solutions initially developed for the 3D incompressible Navier-Stokes equations to other evolution equations that have global solutions which are not known to be unique. We will prove the convergence of statistical solutions of regularized evolution equations to statistical solutions of the original one. The applicability of the theory will be illustrated with the 2D inviscid limit, that is, the convergence of statistical solutions of the 2D Navier-Stokes to the statistical solutions of the 2D Euler equations. This is a joint work with Cecilia Mondaini (Texas A & M) and Ricardo Rosa (UFRJ).

Discrete Bessel functions and partial difference equations

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Abstract

M. Bohner and T. Cuchta have recently proposed a new definition of the discrete Bessel function [1], which is different from the discrete Bessel functions studied in earlier papers. It shares many properties with the classical Bessel function, e.g., it satisfies a difference equation which is a discrete analogue of the Bessel differential equation.

Inspired by this work, we introduce a new class of discrete Bessel functions and discrete modified Bessel functions denoted by $\mathcal{J}_n^c$ and $\mathcal{I}_n^c$ [2]. These functions are the discrete analogues of the functions $t \mapsto J_n(ct)$ and $t \mapsto I_n(ct)$, where I_n and J_n stand for the classical Bessel function and modified Bessel function. If $c = 1$, then $\mathcal{J}_n^c$ reduces to the discrete Bessel function from [1].

Our motivation comes from the theory of lattice differential equations, i.e., equations with discrete space and continuous time. The fundamental solutions of the semidiscrete wave equation have the form $u_1(x,t) = J_{2x}(2ct)$ and $u_2(x,t) = \int_0^t J_{2x}(2cs)\,\mathrm{d}s$, where $x \in \mathbb{Z}$ and $t \geq 0$. The fundamental solution of the semidiscrete diffusion equation has the form $u(x,t) = e^{-2ct} I_x(2ct)$.

Using the new functions $\mathcal{J}_n^c$ and $\mathcal{I}_n^c$, we obtain similar formulas for the fundamental solutions of the purely discrete wave equation and diffusion equation. Formulas for fundamental solutions of these partial difference equations are already available in the existing literature, but in a different form. Expressing them in terms of the discrete Bessel functions can simplify the study of their properties, such as the oscillatory behavior.

References

[1] M. Bohner, T. Cuchta, *The Bessel difference equation*, Proc. Amer. Math. Soc. 145 (2017), 1567–1580.

[2] A. Slavík, *Discrete Bessel functions and partial difference equations*, submitted.

Lebesgue regularity for discrete time nonlocal equations

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Universidad de Santiago de Chile - Chile
Partially funded by FONDECYT 1140258 and and CONICYT - PIA - Anillo ACT1416

Abstract

In this talk, we will present a new method based on operator-valued Fourier multipliers to characterize the existence and uniqueness of ℓ_p-solutions for discrete time fractional models in the form

$$\Delta^\alpha u(n,x) = Au(n,x) + \sum_{j=1}^{k} \beta_j u(n-\tau_j, x) + f(n, u(n,x)), \;\; n \in \mathbb{Z}, x \in \Omega \subset \mathbb{R}^N,$$

where $\beta_j \in \mathbb{R}$, $\tau_j \in \mathbb{Z}$, A is a closed linear operator defined on a Banach space X and Δ^α denotes the Grünwald-Letnikov fractional difference of order $\alpha > 0$. If X is a UMD space, we provide this characterization only in terms of the R-boundedness of the operator-valued symbol associated to the abstract model.

References

[1] Agarwal, Ravi; Cuevas, Claudio; Lizama, Carlos, Regularity of Difference Equations on Banach Spaces, Springer, Cham, 2014.

[2] Lizama, Carlos; Murillo-Arcila, Marina, *Maximal regularity in lp spaces for discrete time fractional shifted equations*, J. Differential Equations. 263 (6) (2017), 3175–3196.

[3] Lizama, Carlos, *ℓ_p-maximal regularity for fractional difference equations on UMD spaces*, Math. Nach., 288 (17/18) (2015), 2079–2092.

Existence and uniqueness of solutions for abstract differential equations with state dependent delay

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Partially supported by Fapesp, Grand 2017/13145-8

Abstract

We present some results on the existence and uniqueness of mild and strict solutions for a general class of differential equations with state dependent delay of the form

$$\begin{aligned} u'(t) &= Au(t) + F(t, u_{\sigma(t,u_t)}), \quad t \in [0,a], && (4)\\ u_0 &= \varphi \in \mathcal{B}_X = C([-p,0];X), && (5) \end{aligned}$$

where $A : D(A) \subset X \to X$ is the generator of an analytic semigroup of bounded linear operators $(T(t))_{t\geq 0}$ defined on a Banach space $(X, \|\cdot\|)$ and $F(\cdot), \sigma(\cdot)$ are suitable continuous functions.

References

[1] Eduardo Hernandez, Michelle Pierri, Jianhong Wu. $\mathbf{C^{1+\alpha}}$-strict solutions and wellposedness of abstract differential equations with state dependent delay. *J. Differential Equations* 261, (2016) 12, 6856-6882.

[2] Eduardo Hernandez, Jianhong Wu. Existence, uniqueness and qualitative properties of global solutions of abstract differential equations with state dependent delay. Submitted.

Dichotomies for generalized ordinary differential equations

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Partially supported by FAPESP grant 2016/24711-1 and CNPq grant 310497/2016-7

Abstract

This talk is concerned with the theory of dichotomies for generalized ordinary differential equations. We study conditions for the existence of exponential dichotomies and bounded solutions. Using the correspondences between generalized ordinary differential equations and other equations, we translate our results to measure differential equations and impulsive differential equations. The fact that we work in the framework of generalized ordinary differential equations allows us to manage functions with many discontinuities and of unbounded variation.

References

[1] E. M. Bonotto, M. Federson and F. L. Santos, Dichotomies for generalized ordinary differential equations and applications. Submitted.

[2] W. A. Coppel, *Dichotomies in Stability Theory*, Lecture Notes in Mathematics, Springer-Verlag, Berlin Heidelberg New York, 1978.

[3] Š. Schwabik, *Generalized Ordinary Differential Equations*, World Scientific, Singapore, Series in real Anal., vol. 5, 1992.

Pullback dynamic in some nonautonomous equations with delay

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Abstract

In this talk we are going to show the well-posedness of the following non-classical and non-autonomous diffusion equation

$$\begin{cases} \dfrac{\partial u}{\partial t} - \gamma(t)\Delta \dfrac{\partial u}{\partial t} - \Delta u = g(u) + f(t, u_t) \text{ in } (s, +\infty) \times \Omega, \\ u = 0 \text{ on } (s, +\infty) \times \partial\Omega \\ u(t,x) = \phi(t-s,x), t \in [s-h, s], x \in \Omega \end{cases} \tag{6}$$

where the term $f(t, u_t)$ denote different kinds of delay. We also study the pullback dynamic of the problem showing the existence of the pullback attractor inside de framework of the evolution processes, assuming some grow conditions for the non-linear term $g(u)$.

References

[1] T. Caraballo, M.A. Márquez-Durán, F. Rivero *Well-Posedness and Asymptotic Behavior of a Nonclassical Nonautonomous Diffusion Equation with Delay.* Int. Journal of Bifurcation and Chaos, Vol. 25, No. 14 (2015) 1540021 (11 pages).

[2] T. Caraballo, M.A. Márquez-Durán, F. Rivero *A Nonclassical and Nonautonomous Diffusion Equation Containing Infinite Delays.* Differential and Difference Equations with Applications, Springer Proceedings in Mathematics and Statistics 164 (2016) 385–399.

[3] T. Caraballo, M.A. Márquez-Durán, F. Rivero *Asymptotic behaviour of a non-classical and non-autonomous diffusion equation containing some hereditary characteristic.* Disc. and Cont. Dynamical System Series B, Volume 22, Number 5 (2017) 1817–1833.

Controllability and Observability for Linear Systems in Banach Spaces using Generalized Ordinary Differential Equations

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Partially supported by Capes

Abstract

It is known that generalized ordinary differential equations (we write generalized ODEs for short), defined by J. Kurzweil, encompass other types of equations such as ordinary and functional differential equations, measure and impulsive differential equations and dynamic equations on time scales. The aim of this paper is to establish results on controllability and observability for a system of linear generalized ODEs defined in a Banach space with initial data, controls and observations also belonging to a Banach space. Necessary and sufficient conditions are obtained. The fact that we work in the framework of generalized ODEs allows us to obtain results for the particular cases where the functions involved can be highly oscillating and have many discontinuities. We apply our results to impulsive differential equation and we point out that similar results are valid for measure differential equations. An example is given to illustrate the results.

References

[1] F. A. Silva, M. Federson, Controllability and Observability for Linear Systems in Banach Spaces using Generalized Ordinary Diffenrential Equations, (2017). Preprint.

Averaging Principle for Neutral Differential Equations

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Partially supported by CAPES

Abstract

The averaging principle is powerful tool in the study of differential equations with some kind of pertubation. However, with respect to the neutral functional differential equations (NFDE) the literature is not very extensive and there are no many results in this way. Here, the purpose is to investigate conditions to establish an averaging principle for these equations based on some ideas of [1],[2].

References

[1] M. Federson and J. G. Mesquita. *Averaging for retarded functional differential equations.* Journal of Mathematical Analysis and Applications **382.1** (2011): 77-85.

[2] R. R. Akhmerov. *On the averaging principle for functional-Differential equations of neutral type.* Ukrainian Mathematical Journal **25.5** (1973): 481-489.

Existence of asymptotically periodic solutions of partial functional differential equations with state-dependent delay

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Abstract

The aim of this work is to establish results about the existence of mild solutions, and the existence of S-asymptotically periodic and asymptotically periodic solutions for systems described by partial or abstract retarded functional differential equations with infinite delay when the delay depends on the state of the system (abbreviated, state-dependent delay) using Lipschitz conditions on the functions involved in the equation. Specifically, in this work we study the existence of asymptotically periodic solutions for a class of abstract retarded functional differential equations (abbreviated, ARFDE) with state-dependent delay described by

$$x'(t) = Ax(t) + f(t, x_{\rho(t,x_t)}), \quad t \in I, \tag{7}$$

$$x_0 = \varphi, \tag{8}$$

where X is a Banach space, $x(t) \in X$, A is the infinitesimal generator of a strongly continuous semigroup of bounded linear operators $(T(t))_{t\geq 0}$ defined on X and f, ρ are functions. In this context, along this work we assume that $f : I \times \mathcal{B} \to X$ is a function such that $f(t, \cdot)$ is continuous for all $t \geq 0$, $\rho : I \times \mathcal{B} \to \mathbb{R}$ is continuous, where $\mathcal{B}$ is the phase space, and f and ρ satisfy additional conditions which will be specified later.

References

[1] F. Andrade ,C. Cuevas, H. R. Henríquez, *Periodic solutions of abstract functional differential equationswith state-dependent delay*, Math. Methods Appl. Sci. 39 (2016), 3897-3909.

[2] H. R. Henríquez, M. Pierri, P. Táboas, *On S-asymptotically ω-periodic functions on Banach spaces and applications*, J. Math. Anal. Appl. 343 (2) (2008), 1119-1130.

[3] H. R. Henríquez, M. Pierri, P. Táboas, *Existence of S-asymptotically ω-periodic solutions for abstract neutral equations*, Bull. Austral. Math. Soc. 78 (2008), 365-382.

[4] J.K. Hale,J. Kato, *Phase space for retarded equations with infinite delay*, Funkcial. Ekvac. 21 (1978), 11-41.

[5] Y. Hino, S. Murakami, T. Naito, *Functional Differential Equations with Infinite Delay*, Lectures Notes in Mathematics 1473, Springer-Verlag, Berlin, 1991.

Non-isothermal model for two viscous incompressible fluids

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Abstract

This talk is concerned with a non-isothermal diffuse-interface model which describes the motion of a mixture of two viscous incompressible fluids. The fluids are assumed to have matched densities and the same viscosity and thermal conductivity. The model consists of modified Navier-Stokes equations coupled with a phase-field equation given by a convective Allen-Cahn equation, and energy transport equation for the temperature. This model is based on an energetic variational formulation. It is investigated the well-posedness of the problem in the two and three dimensional case without any restriction on the size of the initial data. Moreover, regular and singular potentials for the phase-field equation are considered.

A delay differential equation with a solution whose shortened segments are dense

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Abstract

We construct a delay functional $d : Y \to (0, r)$ with $r > 1$, $Y \subset C^1_r = C^1([-r, 0], \mathbb{R})$, and $\dim Y = \infty$ so that the equation

$$x'(t) = -\alpha\, x(t - d(x_t))$$

has a solution whose short segments $x_t|_[- 1, 0]$ are dense in C^1_1. This implies complicated behaviour of the trajectory $t \mapsto x_t \in C^1_r$.

References

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Controllability of nonlinear systems

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Abstract

In this work we are concerned with the controllability of time-varying lumped control systems governed by a nonlinear differential equation. Assuming the underlying linear system is controllable, and the nonlinear forcing function satisfies a boundedness condition which is adapted to the underlying linear system, we show the nonlinear system is approximately controllable.

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On a homogeneous equation related to the KdV equation

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Abstract

In this talk we discuss some properties of the family of equations

$$u_t + 2a\frac{u_x u_{xx}}{u} = \varepsilon a u_{xxx}, \quad (x,t) \in \mathbb{R} \times [0,\infty),$$

where a and ε are arbitrary real parameters. This family of equations was introduced a couple of years ago in the reference [1] where several questions about it were put by the authors, such as: is it somehow related to the KdV equation? Does it have a hierarchy of infinitely many symmetries or conservation laws?

In this talk we shall present positive answers to the questions formulated in [1].

This is a joint work with Dr. P. L. da Silva and Dr. J. C. S. Sampaio.

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L^p-bounded solutions for strongly damped wave equations

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Abstract

We are interested in studying the existence of L^p-bounded solutions to the Cauchy problem

$$\begin{cases} u_{tt} \ + \ 2\eta A^{\frac{1}{2}} u_t \ + \ Au \ = \ f(t,u,u_t), \quad t > 0, \\ u(0) \ = \ u_0 \in X^{\frac{1}{2}}, \ u_t(0) \ = \ v_0 \in X, \end{cases}$$

where $\eta > 0$, X is a reflexive Banach space, $A : D(A) \subseteq X \to X$ is a closed densely defined operator, $X^{\frac{1}{2}}$ is the fractional power space associated with A as in [5] and $f : \mathbb{R}^+ \times X^{\frac{1}{2}} \times X \to X$ is a function given.

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Unbounded Attractors Under Perturbations

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Abstract

We put forward the recently introduced notion of unbounded attractors. These objects will be addressed in the context of a class of 1-D semilinear parabolic equations. The nonlinearities are assumed to be non-dissipative and, in addition, defined in such a way that the equation possesses unbounded solutions as time goes to infinity. Small autonomous and non-autonomous perturbations of these equations will be treated.

Existence and uniqueness of solution for abstract differential equations with state dependent delay at the impulses

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Abstract

We study the existence and uniqueness of mild solutions for a general class of abstract impulsive differential equations with state dependent delay at the impulses.

In this paper we study the existence and uniqueness of mild solutions for a class of abstract impulsive differential equations of the form

$$\begin{aligned} u'(t) &= Au(t) + f(t, u(\zeta(t, u(t)))), \quad t \in I_i = (t_i, t_{i+1}],\ i = 0, \dots, N, & (9)\\ u(t_j^+) &= g_j(u(\sigma_j(u(t_j^+)))), \quad j = 1, \dots, N, & (10)\\ u_0 &= \varphi \in \mathcal{B} = C(I_{-1}; X), \quad I_{-1} = [-p, 0], & (11) \end{aligned}$$

where $A : D(A) \subset X \to X$ is the generator of an analytic semigroup of bounded linear operators $(T(t))_{t\geq 0}$ on a Banach space $(X, \|\cdot\|)$ $0 = t_0 < t_1 < t_2 < \dots < t_N < t_{N+1} = a$ are pre-fixed numbers and $g_i : X \to X$, $f : [0, a] \times X \to X$, $\zeta : [0, a] \times X \to \mathbb{R}$, $\sigma_i : X \to [0, a]$, $i = 1, \dots, N$, are functions that will be specified.

Let $(Z, \|\cdot\|_Z)$ and $(W, \|\cdot\|_W)$ be Banach spaces. We denote by $\mathcal{L}(Z, W)$ the space of bounded linear operators from Z into W endowed with operator norm denoted by $\|\cdot\|_{\mathcal{L}(Z,W)}$. We write $\mathcal{L}(Z)$ and $\|\cdot\|_{\mathcal{L}(Z)}$ if $Z = W$. In addition, $B_r(z, Z) = \{y \in Z : \| y - z \|_Z \leq r\}$.

Let $J \subset \mathbb{R}$ be a bounded interval. The spaces $C(J, Z)$ and $C_{Lip}(J, Z)$ are the usual and their norms are denoted by $\|\cdot\|_{C(J,Z)}$ and $\|\cdot\|_{C_{Lip}(J,Z)}$. We remark that $\|\cdot\|_{C_{Lip}(J;Z)} = \|\cdot\|_{C(J;Z)} + [\cdot]_{C_{Lip}(J;Z)}$ where $[\zeta]_{C_{Lip}(J;Z)} = \sup_{t,s\in J, t\neq s} \frac{\|\zeta(s)-\zeta(t)\|_Z}{|t-s|}$.

The notation $\mathcal{PC}(X)$ is used for the space formed by all functions $u : [0, a] \to X$ such that $u(\cdot)$ is continuous at $t \neq t_i$, $u(t_i^-) = u(t_i)$ and $u(t_i^+)$ exists for all $i = 1, \cdots, n$. This space is provided with the norm $\| u \|_{\mathcal{PC}(X)} = \max_{i=0,1,\dots,N} \| u \|_{C((t_i,t_{i+1}];X)}$. The notation $\mathcal{PC}_{Lip}(X)$ is used for the space of the functions $u \in \mathcal{PC}(X)$ such that $u_{|(t_i,t_{i+1}]} \in C_{Lip}((t_i, t_{i+1}]; X)$ for all $i = 0, 1, \dots N$, endowed with the norm $\| u \|_{\mathcal{PC}_{Lip}(X)} = \max_{i=0,\dots,N} \| u_{|(t_i,t_{i+1}]} \|_{C_{Lip}((t_i,t_{i+1}];X)}$. In addition, we use the notation $\mathcal{BPC}(X)$ (resp. $\mathcal{BPC}_{Lip}(X)$) for the set of functions $u : [-p, a] \to X$ such that $u_0 \in \mathcal{B}$ (resp. $u_0 \in C_{Lip}([-p, 0]; X)$) and $u_{|[0,a]} \in \mathcal{PC}(X)$ (resp. $u_{|[0,a]} \in \mathcal{PC}_{Lip}(X)$).

We introduce the following concept of mild solution.

Definition 1 *A function $u \in \mathcal{BPC}(X)$ is called a mild solution of the problem (17)-(18) if $u_0 = \varphi$, $u(t_i^+) = g_i(u(\sigma_i(u(t_i^+))))$ for all $i = 1, \dots, N$ and*

$$\begin{aligned} u(t) &= T(t)\varphi(0) + \int_0^t T(t-\tau) f(\tau, u(\zeta(\tau, u(\tau))))d\tau, \quad t \in [0, t_1],\\ u(t) &= T(t - t_i)g_i(u(\sigma_i(u(t_i^+)))) + \int_{t_i}^t T(t-\tau) f(\tau, u(\zeta(\tau, u(\tau))))d\tau, \quad t \in (t_i, t_{i+1}]. \end{aligned}$$

To prove our result, we introduce the next conditions.

$\mathbf{H_{Z,\zeta,\sigma_i}}$ $(Z, \| \cdot \|_Z)$ is a Banach space, $\zeta \in C_{Lip}([0,a] \times Z; [-p,a])$, $\sigma_i \in C(Z, [-p,a])$ for all $i \in \{1, \ldots, N\}$, $\sigma_i(x) \neq t_j$ for all $i, j \in \{1, \ldots, N\}$ and there is a function $j : \{1, \ldots, N\} \to \{0, 1, \ldots, N\}$ such that $\zeta \in C_{Lip}(I_i \times Z; I_{j(i)})$ and $j(i) \leq i$ for all $i \in \{1, \ldots, N\}$. Next, for convenience, we write simply $[\zeta]_{C_{Lip}}$ and $[\sigma_i]_{C_{Lip}}$ in place $[\zeta]_{C_{Lip}([0,a]\times V;[-p,a])}$ and $[\sigma_i]_{C_{Lip}(V;[-p,a])}$.

$\mathbf{H^W_{g,Z}}$ $(W, \| \cdot \|_W)$, $(Z, \| \cdot \|_Z)$ are Banach spaces, $(W, \| \cdot \|_W) \hookrightarrow (Z, \| \cdot \|_Z) \hookrightarrow (X, \| \cdot \|)$, $AT(\cdot) \in L^\infty([0,a]; \mathcal{L}(W,Z))$, $g_i \in C_{Lip}(Z; W)$ and $g_i(\cdot)$ is bounded for all $i \in \{1, \ldots, N\}$. Next, $L_{Z,W}(g_i)$ is the Lipschitz constant of $g_i(\cdot)$, $\mathcal{C}_{Z,W}(g_i) = \| g_i \|_{C(Z;W)}$, $L_{Z,W}(g) = \max_{i=\ldots,N} L_{Z,W}(g_i)$ and $\mathcal{C}_{Z,W}(g) = \max_{i=\ldots,N} \mathcal{C}_{Z,W}(g_i)$.

$\mathbf{H^V_{f,Z}}$ $(Z, \| \cdot \|_Z)$, $(V, \| \cdot \|_V)$ are Banach spaces continuously included in $(X, \| \cdot \|)$, $f \in C_{Lip}([0,a] \times Z; V)$ and $f(\cdot)$ is bounded. Next, $L_{Z,V}(f)$ denotes the Lipschitz constant of $f(\cdot)$ and $C_{Z,V}(f) = \| f \|_{C([0,a]\times Z;V)}$.

Notations 1 Next, for convenience, $b = \max\{t_{i+1} - t_i : i = 1, \ldots, N\}$ and $b_i = t_{i+1} - t_i$ for all $i = 1, \ldots, N$. If the conditions $\mathbf{H_{Z,\zeta,\sigma_i}}$, $\mathbf{H^W_{g,Z}}$ and $\mathbf{H^V_{f,Z}}$ are satisfied and $T(\cdot)\varphi(0) \in C_{Lip}([0,a]; V)$, we use the notations

$$\begin{aligned}\Phi_{Z,W,V} &= \| T(\cdot) \|_{L^\infty(([0,b];\mathcal{L}(Z,W))} \mathcal{C}_{Z,W}(g) + \| T(\cdot) \|_{L^\infty([0,b];\mathcal{L}(Z,V))} C_{Z,V}(f) + \Theta_{Z,V} L_{Z,V}(f) \\ &\quad + [T(\cdot)\varphi(0)]_{C_{Lip}([-p,0];Z)} + [\varphi]_{C_{Lip}([-p,0];Z)}, \\ \Theta_{Z,V} &= \| T(\cdot) \|_{L^1([0,b],\mathcal{L}(Z,V))}, \quad \Lambda_{Z,W} = \| T(\cdot) \|_{L^\infty([0,b]\mathcal{L}(Z,W))} .\end{aligned}$$

The main result on the existence of solution for the problem (17)-(18) is given by the following Theorem.

Theorem 1 *Assume that the conditions* $\mathbf{H_{X,\zeta,\sigma_i}}$, $\mathbf{H^W_{g,X}}$ *and* $\mathbf{H^X_{f,X}}$ *are satisfied,* $T(\cdot)\varphi(0) \in C_{Lip}([0,a]; X)$, $\varphi \in C_{Lip}([-p,0]; X)$ *and*

$$\begin{aligned}&2\Theta_{X,X} L_{X,X}(f)(1 + 2[\zeta]_{C_{Lip}}(1 + \Phi_{X,W,X})) \\ &\qquad + 2\Lambda_{X,W} L_{X,W}(g)(1 + 2 \max_{i=1,\ldots,N} [\sigma_i]_{C_{Lip}} \Phi_{X,W,X}) < 1. \end{aligned} \tag{12}$$

Then there exists a unique mild solution $u \in \mathcal{BPC}_{Lip}(X)$ *of the problem (17)-(18).*

We also address the case in which the functions $f(\cdot)$, $g_i(\cdot)$ are unbounded and (or) locally Lipschitz. To begin, we include the next conditions.

$\mathcal{H}^{\mathbf{W}}_{\mathbf{g,Z}}$ $(Z, \| \cdot \|_Z)$, $(W, \| \cdot \|_W)$ are Banach spaces, $(W, \| \cdot \|_W) \hookrightarrow (Z, \| \cdot \|_Z) \hookrightarrow (X, \| \cdot \|)$, $AT(\cdot) \in L^\infty([0,a]; \mathcal{L}(W,Z))$, each function g_i is continuous from Z into W, takes bounded sets into bounded sets and there is $L_{Z,W}(g_i, \cdot) \in C(\mathbb{R}; \mathbb{R})$ such that $\| g_i(x) - g_i(y) \|_W \leq L_{Z,W}(g_i, r) \| x - y \|_Z$ for all $x, y \in B_r(0, Z)$ and every $r > 0$. Next, $L_{Z,W}(g, r) = \max_{i=\ldots,N} L_{Z,W}(g_i, r)$ and $\mathcal{C}_{Z,W}(g_i, r) = \| g_i \|_{C(B_r(0,Z);W)}$.

$\mathcal{H}^{\mathbf{V}}_{\mathbf{f,Z}}$ $(Z, \| \cdot \|_Z)$, $(V, \| \cdot \|_V)$ are Banach spaces, $f(\cdot)$ is continuous from Z into V, takes bounded sets into bounded sets and there is a function $\mathcal{L}_{Z,V}(f, \cdot) \in C(\mathbb{R}; \mathbb{R})$ such that $\| f(x) - f(y) \|_V \leq \mathcal{L}_{Z,V}(f, r) \| x - y \|_Z$ for all $x, y \in B_r(0, Z)$ and every $r > 0$. Next, for $r > 0$ we use the notation $\mathcal{C}_{Z,V}(f, r) = \| f \|_{C([0,a]\times B_r(0,Z);V)}$.

Notations 2 If the above conditions are satisfied, for $r > 0$ we define $\Phi_{Z,W,V}(r)$ by

$$\begin{aligned}\Phi_{Z,W,V}(r) &= \| T(\cdot) \|_{L^\infty(([0,b];\mathcal{L}(Z,W))} \mathcal{C}_{Z,W}(g, r) + \| T(\cdot) \|_{L^\infty([0,b];\mathcal{L}(Z,V))} C_{Z,V}(f, r) \\ &\quad + \Theta_{Z,V} L_{Z,V}(f, r) + [T(\cdot)\varphi(0)]_{C_{Lip}([-p,0];Z)} + [\varphi]_{C_{Lip}([-p,0];Z)}.\end{aligned}$$

Proposition 1 *Let the conditions* $\mathbf{H}_{\mathbf{X},\zeta,\sigma_{\mathbf{i}}}$, $\mathcal{H}_{\mathbf{g},\mathbf{X}}^{\mathbf{W}}$ *and* $\mathcal{H}_{\mathbf{f},\mathbf{X}}^{\mathbf{X}}$ *be holds. Suppose that* $T(\cdot)\varphi(0) \in C_{Lip}([0,a];X)$ *and there is* $r > \| \varphi \|_{C([-p,0];X)}$ *such that the inequality (12) is satisfied with* $L_{X,X}(f,r), \Phi_{X,W,X}(r$ *and* $L_{X,W}(g,r)$ *in place* $L_{X,X}(f), \Phi_{X,W,X}$ *and* $L_{X,W}(g)$, *and*

$$C_0 \| \varphi(0) \| + \| T(\cdot) \|_{L^\infty(([0,b];\mathcal{L}(W,X)} \mathcal{C}_{X,W}(g,r) + \| T(\cdot) \|_{L^1([0,b];\mathcal{L}(X))} \mathcal{C}_{X,X}(f,r) < r.$$

Then there exists a unique mild solution $u \in \mathcal{BPC}_{Lip}(X)$ *of (17)-(18).*

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Non-absolute integration: pros and cons... and more pros

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Abstract

We discuss the main properties, virtues as well as weaknesses, of the non-absolute integration theory due to Jaroslav Kurzweil and Ralph Henstock which gives rise to the so-called generalized Riemann integral.

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Nonlocal problems in perforated domains

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Abstract

In this talk we analyze nonlocal problems of the form

$$f(x) = \int_B J(x-y)(u(y)-u(x))dy$$

with x in a perforated domain $\Omega^\epsilon \subset \Omega$. Here J is a non-singular kernel. We think about Ω^ϵ as a fixed set Ω from where we have removed a subset that we call the holes. We deal both with the Neumann and Dirichlet conditions in the holes and assume a Dirichlet condition outside Ω. In the later case we impose that u vanishes in the holes but integrate in the whole $\mathbb{R}^N$ ($B = \mathbb{R}^N$) and in the former we just consider integrals in $\mathbb{R}^N$ minus the holes ($B = \mathbb{R}^N \setminus (\Omega \setminus \Omega^\epsilon)$). Assuming weak convergence of the holes, specifically, under the assumption that the characteristic function of Ω^ϵ has a weak limit, $\chi_\epsilon \rightharpoonup \mathcal{X}$ weakly* in $L^\infty(\Omega)$, we analyze the limit as $\epsilon \to 0$ of the solutions to the nonlocal problems proving that there is a nonlocal limit problem. In the case in which the holes are periodically removed balls we obtain that the critical radius is of order of the size of the typical cell (that gives the period). In addition, in this periodic case, we also study the behavior of these nonlocal problems when we rescale the kernel in order to approximate local PDE problems.

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Computational and Numerical Analysis of a Nonlinear Mechanical System with Bounded Delay

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Abstract

Modern structures are increasingly resistant and complex. In many cases, such systems are modeled by numerical approximations methods, due to its complexities. The study of vibration levels in the response of a system is of great importance to have a reliable and efficient design, since those vibrations are undesirable phenomena that may cause damage, failure, and sometimes destruction of machines and structures. In this paper is investigated the modeling strategy of nonlinear system with damping, subject the time delayed. Focuses on the theoretical study and numerical simulations of a two degree-of-freedom nonlinear damped system, constituted of a primary mass attached to the ground by a spring and damping, with linear or nonlinear characteristics (primary system) , and the secondary mass attached to the primary system by a spring and damping with linear or nonlinear characteristics (Secondary system), for the integration of equations of motion will be used Fourth Order Runge-Kutta Method. The behavior of a nonlinear main system with nonlinear secondary system will be investigated to many cases of resonances. In this case, we used are various delay time values for confirming its influence of the attenuation of vibrations, but, unfortunately, also in increasing the nonlinearity (instable responses) of the system in question.

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Atmospheric Dust Dynamic Modeled by a Wavelength-Fractional Diffusion Equation

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Abstract

To achieve information about the measure of the amount of solar radiation at the Martian surface is useful to gain some insight into the following issues:

- UV irradiation levels at the bottom of the Martian atmosphere to use them as an habitability index.
- Incoming shortwave radiation and solar heating at the surface.
- Relative local index of dust in the atmosphere.

The obtention of these data is affected by the different Martian atmospheric scenarios. In particular, the dust aerosols have an important effect on the solar radiation in the Martian atmosphere and both surface and atmospheric heating rates, which are also basic drivers of atmospheric dynamics [1]-[3].

Aerosols cause an attenuation of the solar radiation traversing the atmosphere and this attenuation is modeled by the Lambert-Beer-Bouguer law,

$$F(\lambda) = DF_0(\lambda)e^{-\tau(\lambda)m},$$

where $F_0(\lambda)$ is the spectral irradiance at the top of the atmosphere, m is the absolute air mass, D is the correction factor for the earth-sun distance, and $\tau(\lambda)$ is the total optical thickness at wavelength λ, in which the aerosol optical thickness $\tau_a(\lambda)$ plays an important role. Through Angstrom law, the aerosol optical thickness can be approximated as a second order moment,

$$\tau_a^{-1}(\lambda) = \frac{\lambda^\alpha}{\beta},$$

where, α, β are parameters related to the dust particles and the properties of the atmosphere, and then this law allows to model attenuation of the solar radiation traversing the atmosphere by a wavelength-fractional diffusion equation [4]-[7]:

$$\frac{\partial^\alpha \varphi}{\partial \lambda^\alpha} = \frac{\Gamma(\alpha+1)}{2\beta} \frac{\partial^2 \varphi}{\partial x^2}, \qquad 0 < \alpha < 2.$$

The analytical solution of the fractional diffusion equation is available in the case of one space dimension and three space dimensions with radial symmetry. When we extend the fractional diffusion equation to the case of two or more space variables, we need large and massive computations to approach the solutions through numerical schemes. In this case a suitable strategy is to use the cloud computing to carry out the simulations.

In this study, we discuss some questions of the model and experimental data. We present analytic solutions for this modeling problem in one and three space dimensions and numerical methods that allow us to obtain computational simulations of the solutions. Also, the fractional model provides information that can be understood in term of higher order moments and this relation establishes a meeting point and discussion regarding to the experiments. In this context, we are working in the fitting of the fractional model to dust observational data [8].

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Oscillation and nonoscillation criteria for impulsive delay differential equations with Perron integrable righthand sides

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Abstract

Let $p : [t_0, \infty) \to \mathbb{R}$ and $g : [t_0, \infty) \to \mathbb{R}$. Consider the measure delay differential equation with impulses

$$\begin{cases} Dy = -p(t)y(t-\tau)Dg \\ y(t_k^+) - y(t_k) = b_k y(t_k), \quad k \in \mathbb{N}, \end{cases} \tag{13}$$

in which g is a regulated function which is left-continuous and continuous at the points of impulses t_k, $k \in \mathbb{N}$, Dy and Dg stand for the distributional derivatives of the functions y and g in the sense of distributions of L. Schwartz and, moreover,

- $t_0 < t_1 < \ldots < t_k < \ldots$ are fixed points and $\lim\limits_{k\to\infty} t_k = \infty$;
- for $k \in \mathbb{N}$, $b_k \in (-\infty, -1) \cup (-1, \infty)$ are constants;
- for each compact subset $[a, b]$ of $[t_0, \infty)$ the Perron-Stieltjes integral $\displaystyle\int_a^b p(s)dg(s)$ exists.

The objective of this work is to present new criteria for the existence of oscillatory and nonoscillatory solutions of equation (1).

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Impulsive Autonomous Systems

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Abstract

This is a joint work with Prof Miguel V.S. Frasson, Prof Selma H.J. Nicola and Prof Plácido Z. Táboas. Our object of study is an autonomous ODE submitted to an impulsive self-support and occasionally to an initial condition,

$$\dot{x} = f(x), \tag{14}$$

$$x(t) \in M \implies x(t+) = F(x(t)), \tag{15}$$

$$x(t_0) = b. \tag{16}$$

where $x \in \mathbb{R}^n$, $f \in C^1$, $M \subset \mathbb{R}^n$ is closed and $F : M \to \mathbb{R}^n$ is continuous. We define a topological approach by identifying a point $x(t) \in M$ with $F(x(t))$, so that we eliminate the impulse at $x(t)$. This process in general leads a discontinuous semi-dynamical system to a continuous one in a more complex topological space.

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Higher-Order Beverton–Holt Equations

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Abstract

This is joint work with Fozi Dannan and Sabrina Streipert. In this talk, we discuss a certain nonautonomous Beverton–Holt equation of higher order. After an introduction to the classical Beverton–Holt equation and recent results, we solve the higher-order Beverton–Holt equation by rewriting the recurrence relation as a difference system of order one. In this process, we examine the existence and uniqueness of a periodic solution and its global attractivity. We continue our analysis by studying the corresponding second Cushing–Henson conjecture, i.e., by relating the average of the unique periodic solution to the average of the carrying capacity.

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Recent results on impulsive systems

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Abstract

Impulsive dynamical systems describes evolution of systems where the continuous development of an evolution is interrupted by abrupt changes of state, which we call *impulses* (or *corrections*). Such subject has been the research topic of many authors over the last five decades and many real world problems can be defined in terms of impulsive systems; for instance, a simple medicine intake, which requires that a new dose must be taken in order to keep the disease under control, and hence the concentration of the medicine suffers a sudden change.

In this talk I will present a brief history of this theory and many recent results regarding these systems, for both the autonomous and nonautonomous cases. We will see the basic definitions, important properties, results on the existence of 'attractors' and the relationship among them. Moreover, I will present some of the difficulties encountered and some open problems.

This overview is a collection of results from several papers (see [1, 2, 3, 4, 5]) in collaboration with Alexandre Carvalho (ICMC - USP), Everaldo Bonotto (ICMC - USP), Radosław Czaja (University of Silesia, Katowice - Poland), Rodolfo Collegari (UFU) and Tomás Caraballo (University of Seville, Spain).

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On a wave equation with degenerate memory

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Abstract

This paper is concerned with the long-time dynamics of a semilinear wave equation with degenerate viscoelasticity

$$u_{tt} - \Delta u + \int_{-\infty}^{t} g(t-s) div[a(x)\nabla u(s)]ds + f(u) = h(x),$$

defined in a bounded domain Ω of $\mathbb{R}^3$, with Dirichlet boundary condition and nonlinear forcing $f(u)$ with critical growth. The problem is degenerate in the sense that the function $a(x) \geq 0$ in the memory term is allowed to vanish in a part of $\overline{\Omega}$. When $a(x)$ does not degenerate and g decays exponentially it is well-known that the corresponding dynamical system has a global attractor without any extra dissipation. In the present work we consider the degenerate case and prove the existence of global attractors by adding a complementary frictional damping $b(x)u_t$, which is in certain sense arbitrarily small, such that $a + b > 0$ in $\overline{\Omega}$.

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Measure Neutral Functional Differential Equations with infinite delay and Generalised Ordinary Differential Equations

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Abstract

In order to study Neutral Functional Differential Equations with infinite delay, we stablish a equivalence of a class of these equations with a class of Generalised Ordinary Differential Equations. Results on existence, uniqueness of solutions and continuous dependence with respect to initial conditions are obtained.

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Navier-Stokes equations with Caputo time fractional derivative

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Abstract

The aim of this lecture is to analyze the generalized Navier-Stokes equations with time fractional differential operator:

$$\begin{aligned} cD_t^{\alpha}u - \nu\Delta u + (u\cdot\nabla)u + \nabla p &= f \qquad \text{in } \mathbb{R}^N,\ t>0,\\ \nabla\cdot u &= 0 \qquad \text{in } \mathbb{R}^N,\ t>0,\\ u(x,0) &= u_0 \qquad \text{in } \mathbb{R}^N, \end{aligned}$$

where cD_t^{α} is the Caputo fractional derivative of order $\alpha\in(0,1)$ and f a suitable function. More specifically, we address this matter using the theory of fractional abstract Cauchy problems, proving that it possesses an unique global mild solution with certain interesting decay properties.

Then we discuss the integrability in time of this solution and show that it has a non expected regularity. Finally, we use all the obtained information to guess some properties of the classical solution.

This is a joint work with Prof. Gabriela Planas from UNICAMP.

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A Singular and Shear Wave Packet in a Rapidly Rotating Vortex

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Abstract

This study describes the asymptotic quasi-steady régime attained by a rapidly rotating vortex after a wave packet has interacted with it. We consider singular nonlinear helical and shear modes within a linearly stable, columnar, axisymmetric, and dry vortex in the f-plane. The presence of asymmetric disturbances inside a vortex is a possible intensification mechanism. The normal modes enter resonance with the vortex at a certain radius r_c, where the phase angular speed is equal to the rotation frequency. The singularity in the modal equation at r_c strongly modifies the flow in the 3D helical critical layer, the region where the wave/vortex interaction occurs. For a $O(\varepsilon)$ small amplitude wave packet, this interaction induces a secondary mean flow of $O(\varepsilon^{1/2})$ amplitude that can be observed in tropical cyclones in the form of inner spiral bands[2]. The outcome is that the wave/vortex interaction is all the stronger as the wave packet is localized. For a $O(\varepsilon^{1/2})$ packet vertical extent, the cat's eye loses its symmetry with respect to the radial axis owing to the presence of a $O(\varepsilon)$ mean radial circulation. Matched asymptotic analysis shows that two slow times are involved, and that neutral modes are distorted. We derive the system of nonlinear coupled PDEs that governs the slowly evolving amplitudes of the wavepacket and induced mean flow in the quasi-steady régime. The nonlinear terms of the integro-differential evolution equation of the wave packet amplitude are related to the deformed shape of the cat's eye and to the distortion of the mean axial vorticity. This system leads to a more complex dynamics with respect to the previous studies on wavepackets where the coupling was omitted [1] and where a Korteveg-de-Vries equation was derived.

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Event horizons and global attractors

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Abstract

The Einstein constraint equations describe the space of initial data for the evolution equations, dictating how space should curve within spacetime. Under certain assumptions, the constraints reduce to a scalar quasilinear parabolic equation on the sphere with various singularities, and nonlinearity being the prescribed scalar curvature of space. We focus on self-similar solutions of Schwarzschild type. Those describe, for example, the initial data of black holes. In this case, we show that the event horizon is related with global attractors of such parabolic equations. Lastly, some properties of those attractors and its solutions are explored.

Convergence of attractors for a nonautonomously and hyperbolically perturbed semilinear parabolic equation

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Abstract

We consider the initial and boundary value problem governed by the equation $u_t - \Delta u = f_0(u)$ on a bounded domain $\Omega \subset \mathbb{R}^3$ with the homogeneous Dirchlet conditions and cubic nonlinearity f_0. We compare the global attractor of the semiflow governed by the above equation with uniform, pullback, and cocycle attractors of the process governed by its nonautonomous perturbation $\epsilon u_{tt} + u_t - \Delta u = f_\epsilon(t, u)$, where the type of equation changes from parabolic to hyperbolic. Under appropriate conditions on convergence of f_ϵ to f_0 we prove that all three types of nonautonomous attractors converge in the sense of Hausorff distance, both upper- and lower-semicontinuously, to the global attractor for the unperturbed problem as $\epsilon \to 0$. The problem has application in modelling of the heat processes with the Fourier law replaced by the Maxwell-Cattaneo law. This is joint work with José A. Langa (Universidad de Sevilla, Spain) and Mirelson M. Freitas (Universidade Federal do Pará, Brazil).

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Generalized semiflows for a plate model with nonlinear strain

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Abstract

In this talk we are concerned with the long-time dynamics of a class of plate equations, with nonlinear strain of p-Laplacian type,

$$u_{tt} + Au + A^{\alpha}u_t - div(|\nabla u|^{p-2}\nabla u) + f(u) = h(x),$$

with $A = \Delta^2$, $\alpha \in (0, \frac{1}{2})$, $p \geq 0$, and defined in a bounded domain of $\mathbb{R}^2$. This kind of equation was studied by many authors. However, without a strong damping term $-\Delta u_t$, the uniqueness of the problem is known only for some special cases. See for instance [2]. Then we study this problem by using the theory of generalized semiflows [1], which is dedicated to evolution problems with presumed non-uniqueness.

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Semicontinuity of attractors for impulsive dynamical systems

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Abstract

In this work we introduce the concept of collective tube conditions which assures a suitable behavior for a family of dynamical systems close to impulsive sets. Using the collective tube conditions, we develop the theory of upper semicontinuity of global attractors for a family of impulsive dynamical systems.

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Well posedness, regularity, and asymptotic behavior of linear fractional integro–differential equations with order varying in time

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Abstract

The well posedness of abstract time evolution fractional integro–differential equation with variable order $u(t) = u_0 + \partial^{-\alpha(t)} Au(t) + f(t)$, as well as the asymptotic behavior as $t \to +\infty$, and the regularity of its solutions are studied. Here A plays the role of a linear operator of sectorial type.

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Remark on autonomous generalized ordinary differential equations

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Abstract

In this talk, we are going to present the autonomous generalized ODEs and show that these equations do not enlarge the class of the classical autonomous ODEs, even when we consider a more general class of functions in the right-hand sides of the equation.

Contiguity of States and Super Wave Operators for Quantum Markov Semigroups

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Abstract

The qualitative analysis of Quantum Markov Semigroups (QMS) has been strongly influenced by both, Quantum Mechanics as well as Semigroup Theory in Probability. For instance, in my old paper [6] I analyzed a number of notions inspired by Scattering Theory, in particular, the existence of the so-called wave operators. The key was an idea due to Lucien Le Cam, who introduced in classical probability the notion of contiguity. He dedicated his life to develop Mathematical Statistics at a very high level, including subtle results in Functional Analysis as well. The conference is aimed at providing a panorama of recent results obtained by defining a concept of contiguity of states on von Neumann's algebras. As a result, one obtains a characterization of a Super Wave Operator (SWO) for two given open quantum dynamics. These results will be illustrated via quantum and classical examples of evolutions.

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Blow-up of solutions of semilinear heat equations at the almost Hénon critical exponent

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Abstract

We study the problem

$$\begin{cases} u_t - \Delta u = |x|^{\alpha}|u|^{\frac{4+2\alpha}{N-2}-\varepsilon}u & \text{in } B_1 \times (0,\infty) \\ u = 0 & \text{on } \partial B_1 \times (0,\infty) \\ u = u_0 & \text{in } B_1 \times \{0\}, \end{cases} \tag{P_ε}$$

where B_1 is the unit ball in $\mathbb{R}^N$, $N > 2$, $\varepsilon > 0$ is an small parameter, and $\alpha > 0$ is a real number which is not an even integer. We show that if $\varepsilon > 0$ is small enough, then there exists a sign-changing stationary solution ψ_ε of (P_ε) such that the solution of (P_ε) with initial value $u_0 = \lambda\psi_\varepsilon$ blows up in finite time if $|\lambda - 1| > 0$ is sufficiently small.

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Stability for a Dynamic Equation in Time Scales

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Abstract

This talk is devoted to the stability of a second order dynamic equation in Time Scale (see [4] for information) with delayed argument

$$x^{\Delta\Delta}(t) + b(t)x^{\Delta}(g(t)) + c(t)x(h(t)) = f(t),\ t \in \mathbb{T},$$

where $\mathbb{T}$ is a non upper bounded time scale and $b, c, g, h : \mathbb{T} \to \mathbb{T}$ are nonnegative functions such that $h(t), g(t) \leq t$. Particularly, we study common element between autonomous case [6], non autonomous case [2, 3] and unification of discrete and continuous results [5]. We make comparison with the linear version with the results given by [1, 7].

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Recent results on the stability of 2D-Navier-Stokes equations with unbounded delay

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Abstract

In this talk we exhibit different methods to analyze the asymptotic behavior of solutions to a 2D-Navier-Stokes model when the external force contains hereditary characteristics (constant, distributed or variable delay, memory, etc). First we recall some results on the existence and uniqueness of solutions. Next, the existence of stationary solution is established by Lax-Milgram theorem and Schauder fixed point theorem. Then the local stability analysis of stationary solution is studied by using the theory of Lyapunov functions, the Razumikhin-Lyapunov technique. In the end, Lyapunov functionals is also exploited some stability results. We highlight the differences in the asymptotic behavior in the particular case of bounded or unbounded variable delay.

Existence and uniqueness of solutions for second order abstract impulsive differential equations with state-dependent delay at the impulses

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Abstract

We consider a second order abstract differential equations with state-dependent delay at the impulses of the form

$$\begin{aligned} u''(t) &= Au(t) + f(t, u(t), u'(t)), \quad t \in [0, a],\, t \neq t_i, \\ u_0 &= \varphi \in C([-p, 0]; X), \\ u'(0^+) &= x \in X, \\ \triangle u(t_i) &= J_i(u(\sigma(u(t_i)))), \end{aligned}$$

where A is the generator of a strongly continuous cosine function of bounded linear operators $(C(t))_{t\in\mathbb{R}}$ on a Banach space $(X, \|\cdot\|)$; $0 = t_0 < t_1 < ... < t_N = a$ are pre-fixed numbers and $\triangle u(t_i)$ represents the jump of the function u at t_i, which is defined by the function $J_i : X \to X$; $f : [0, a] \times X \times X \to X$ and $\sigma : X \to \mathbb{R}$ are specified functions. We study existence and uniqueness of solutions for this class of equations and present an example related to partial differential equations with state dependent delay at the impulses.

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Abstracts of the Posters

The Poisson Equation: Application to Physics

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Abstract

The partial differential equations are of extreme importance in physics because they describe physical phenomena whose behaviour depends on the position, such as electrostatic, electrodynamic, electromagnetism, fluid dynamics, heat diffusion, wave propagation, etc. These equations are classified in *hyperbolic, parabolic* or *elliptical.*

The Poisson equation is an elliptical equation of partial derivatives in the form

$$\Delta u = \sum_{i=1}^{n} \frac{\partial^2 u}{\partial x_i^2} = f$$

where $f : U \subset \mathbb{R}^n \mapsto \mathbb{R}$ and Δ denotes Laplace's operator, with extensive utility in Electrostatica, stationary models, such as heat equation, fluid dynamics, etc.

In this work we will show the proper importance of PDE's and resolve the Poisson equation by the method of separating variables in a physical application such as the tension function of a bar or electric power.

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IDE: Optimization problems and population dynamics

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Abstract

In this paper, after an introduction to Impulsive Differential Equations (IDE), the article "Optimization problems for one-impulsive models from population dynamics", studied by the end of the scientific initiation project, will be presented. This article introduces some optimization results to be applied in population dynamics models involving first-order impulsive differential equations.

The population dynamics models are represented by the ordinary equation

$$\frac{dN}{dt} = f(t, N), \tag{17}$$

where $N = N(t) > 0$ is the population size in time $t \geq 0$ and $f(t, N)$ is the total growth rate of the population size. Besides that, in so many cases, $f(t, N) = f(N)$, then we say that there is a temporal constancy of the environment, and the ODE (17) is said to be homogeneous.

Depending on the choice of f and the characteristics of the environment and studied population, we obtain different different ODE models. On this paper, the Logistic and Gompertz equations will be considered.

Our main objective is to construct an impulsive problem as follows:

$$\begin{cases} \eta'(t) = f(t, \eta),\ t \neq \tau,\ t \in [0, T]\,, \\ \Delta\eta(\tau) = \eta(\tau^+) - \eta(\tau^-) = -I\,, \\ \eta(0) = \eta_0\,, \end{cases} \tag{18}$$

with $I > 0$ and impulse moment $0 < \tau \leq T$.

From the considered hypotheses, we can guarantee that, for each t fixed in the given interval $[0, T]$, exists an only value $\psi(t)$ such that

$$f(t, \psi(t)) - f(t, \psi(t) - I) = 0\,.$$

Also, ψ is continuous in $[0, T]$ and $\psi(t) \in (M(t), M(t) + I)$ for all t in this interval.

Thus, if the solution $N(t;0,N_0)$ of the PVI given by the equation (17) meets the function ψ in the instant τ, then the solution of (18) satisfies $f(\tau,\eta(\tau)) - f(\tau,\eta(\tau) - I) = 0$, that is, the growth rate doesn't change in the impulse moment. This fact is very important to the optimization results.

Another result says that the set of instants in which the solution of (17) meets the funcion ψ consists of not more than one point. Therefore, the impulse moment in (18) will be choses as the instant τ such that $N(\tau) = \psi(\tau)$.

On the other hand, the main theorems present the optimization results. They guarantee that the solution with impulse moment τ such that $N(\tau) = \psi(\tau)$ assume bigger values in the instant T than the solution of the same equation with any different impulse moment $\hat{\tau} \neq \tau$.

Consequently, applying the results to the population dynamic models, it's possible to construct impulsive problems with more satisfactory optimization results, so that an applied study brings desired results.

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Expansão de autofunções para problemas de Sturm-Liouville com condições de transmissão num ponto interior

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Abstract

O propsito do artigo que ser apresentado na modalidade exposio de pster extender algumas propriedades do problema regular de Sturm-Liouville tipos especiais de problemas de fronteira descontnuas, os quais consiste da equao de Sturm-Liouville junto com condies de fronteira eigenparameter-dependent e duas condies de transmisso suplementares. Ns construmos o operador resolvente e a funo de Green e provamos teoremas sobre expanso em termos de autofunes em espaos de Hilbert $L_2\left[a, b\right]$ modificado.

Investigamos algumas propriedades espectrais de um dos problemas de Sturm-Liouville descontnuos para o qual o parmetro autovalor aparece tanto na equao diferencial quanto nas condies de fronteira. Alm disso duas condies de transmisso suplementares no ponto interior so adicionadas as condies de fronteira. Mais precisamente, iremos considerar a equao de Sturm-Liouville

$$\tau u := -u''(x) + q(x)u(x) = \lambda u(x)$$

mantendo o invervalo finito (a, b) exceto num ponto interior $c \in (a, b)$, onde a descontinuidade em u e u' so prescritas pelas condies de transmisso

$$\gamma_1 u(c-0) - \delta_1 u(c+0) = 0,$$

$$\gamma_2 u'(c-0) - \delta_2 u'(c+0) = 0,$$

junto com as condies de fronteira eigenparameter-dependent

$$\alpha_1 u(a) + \alpha_2 u'(a) = 0,$$

$$\left(\beta'_1 \lambda + \beta_1\right) u(b) - \left(\beta'_2 \lambda + \beta_2\right) u'(b) = 0,$$

onde o potencial $q(x)$ um valor real, contnuo em cada intervalo $[a, c)$ e $(c, b]$ e possui limites finitos $q(c\mp 0); \alpha_i, \beta_i, \beta'_i, \delta_i, \gamma_i \, (i = 1, 2)$ so nmeros reais; λ um eigenparameter complexo. Naturalmente, exclumos cada uma das condies triviais $\gamma_1 = \delta_1 = 0$, $\gamma_2 = \delta_2 = 0$, $\alpha_1 = \alpha_2 = 0$, $\beta'_1 = \beta_1 = \beta'_2 = \beta_2 = 0$. Autofunes deste problema podem ter descontinuidades num ponto interior do intervalo considerado. Este tipo de problema est relacionado com descontinuidades de propriedades materiais, como calor, transferncia de massa, vrias classificaes de problemas de transferncia fsicos, problemas de vibrao da corda onde a corda carregada adicionalmente com pontos de massa e prolemas de difrao.

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Population dynamics models

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Abstract

The ordinary differential equations are generally used in the modeling of problem envolving population dynamics, where we assume the system is governed by the principle of causality. But more realistic models should include some of the past state of this system. So the aim of this work is to develop the basic theory about delay differential equations, present an aplication using the Hutchinson's equation

$$\dot{x}(t) = rx(t)[1 - x(t-\tau)/K], \tag{19}$$

that can be rewritten as

$$\dot{y}(t) = -\alpha y(t-1)[1 + y(t)], \tag{20}$$

and then show some of the differences between ordinary differential equations (ODEs) and retarded functional differential equations (RFDEs) in population dynamics models.

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Zero-one law for α-resolvent families

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Abstract

In this work we consider α-resolvent families $S_\alpha(t)$, $\alpha > 0$, $t \geq 0$, studied by Bazhlekova [1], and prove a zero-one law for this families.

Theorem 2 *Let $(S_\alpha(t))_{t\geq 0}$ be a α-resolvent family generated by A. Suppose that*

$$\sup_{t\geq 0} \|S_\alpha(t) - I\| =: \theta < 1.$$

Then $S_\alpha(t) = I$ for all $t \geq 0$.

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Characterization of the dual space of C([a,b])

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Abstract

In this work we will characterize the dual space of the continuous functions from $[a, b]$ to $\mathbb{R}$, denoted by $C([a, b])$. For this purpose, we will use the Riesz's Theorem and the space of the normalized functions of bounded variation, $NBV([a, b])$, that is a subspace of the functions of bounded variation, $BV([a, b])$. Finally, we will see that there is a homeomorphism between $(C([a, b]))'$ and $NBV([a, b])$.

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Henstock-Kurzweil Integral and the Kurzweil equations

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Abstract

In this work, we investigate the properties of the Henstock-Kurzweil integral and we present a comparison between this integral and the classic Riemann integral. Also, we study the generalized ordinary differential equations (GODEs) and their properties. Finally, we present a relation between the solutions of the GODEs and the solutions of the ordinary differential equations.

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Relations among the Kurzweil-Henstock, Lebesgue and McShane integrals

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Abstract

In this work we present the main theorems connecting the Lebesgue, Kurzweil-Henstock and McShane integrals. It is shown that the complicated Lebesgue integral can be seen as a particular case of the Kurzweil integral, which can be defined in a much simpler way. We also show the equivalence between the Lebesgue integral and the McShane integral, which is a small variation of the Kurzweil integral.

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Asymptotic behavior of Sobolev type resolvents and its applications

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Abstract

This talk shows the asymptotic behavior of resolvent operators of Sobolev type and its applications to the existence and uniqueness of mild solutions to fractional functional evolution equations of Sobolev type in Banach spaces. We first study the asymptotic decay of some resolvent operators (also called solution operators) and next, by using fixed point theorems, we obtain the existence and uniqueness of solutions to a class of Sobolev type fractional differential equation.

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